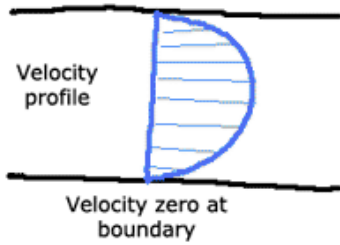


FLUID FLOW

Thursday, 18 August 2011
10:34 AM

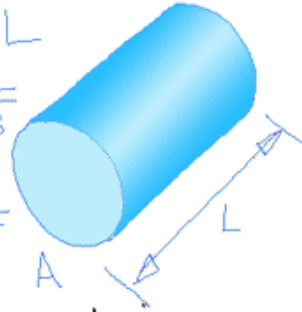
FLUID FLOW: 20100817 cH11 P 235



$$V = AL$$

$$v = \frac{L}{s}$$

$$\dot{V} = \frac{V}{s} = \frac{AL}{s}$$



per second

Fluid Flow

$$\dot{V} = vA$$

\dot{V} volume flow rate
 m^3/s m/s m^2

Mass Flow rate $\dot{m} = \frac{m}{t} = vA\rho$

kg/s

\dot{m} = Mass flow rate

Continuity: IN = OUT

Fluid Flow (p235)



Q2: (cont) The Trans-Alaska oil pipeline carries 2.14 million barrels of crude oil ($\rho = 0.82$) in one day. (b) What is the mass flowrate? (1 barrel = 117.3478 litres)

117.3478 litres per barrel

2.14E6 per day

$$\begin{aligned} \text{Volume} &= 2.14\text{E}6 * 117.3478 = 251124292 \text{ L/day} \\ &= 251124292 / 1000 = 251124.292 \text{ m}^3/\text{day} \end{aligned}$$

$$\text{Per second} = 251124.292 / (60 * 60 * 24) = 2.9065 \text{ m}^3/\text{s}$$

$$\begin{aligned} \text{Mass flow rate: } \rho &= m / \dot{V} \text{ so } m = \rho * \dot{V} \\ \dot{m} &= 0.82 * 1000 * 2.9065 = 2383.33 \text{ kg/s} \end{aligned}$$

$$\text{OIL} = 820 \text{ kg/m}^3$$

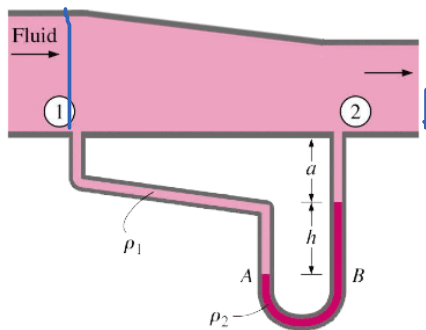
Diam = 1220: Find velocity of oil.

$$A = \pi * 1.22^2 / 4 = 1.16898 \text{ m}^2$$

$$\dot{V} = vA$$

$$v = \dot{V} / A = 2.9065 / 1.16898 = 2.48636 \text{ m/s}$$

Continuity Flow (p237)



Q4: Water flows at 3.5 m/s in a pipe of diameter 113 mm. The pipe transitions down to 55 mm diameter. What is the velocity in the smaller pipe?

From continuity

$$\dot{V}_1 = \dot{V}_2$$

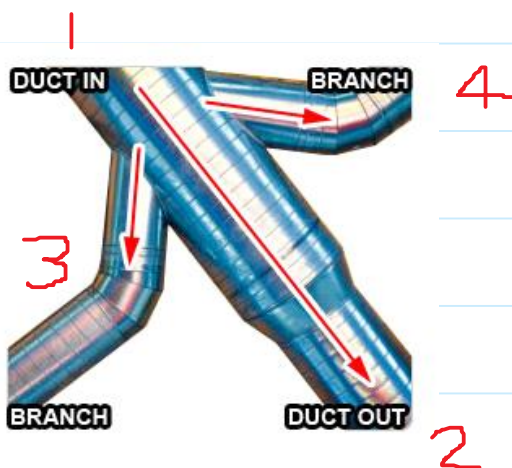
$$v_1 A_1 = v_2 A_2$$

$$v_2 = v_1 A_1 / A_2$$

$$= 3.5 * (\text{Pi} * 0.113^2 / 4) / (\text{Pi} * 0.055^2 / 4)$$

$$= 14.7740 \text{ m/s}$$

A(0.113)



Q9: Air (Spec vol 0.89 m³/kg) flows at 7.1 m/s in a duct of 315 mm diam. After 2 branches of diam 142 mm the flow is now 5.8 m/s and the duct reduces to 268 mm diam. (a) Find total mass flow rate.

$$\text{Spec Vol} = 1 / \rho$$

$$\rho = 1 / 0.89 = 1.1236 \text{ kg/m}^3$$

$$\dot{V}_1 = v A = 7.1 * \text{Pi} * 0.315^2 / 4 = \underline{0.55331 \text{ m}^3/\text{s}}$$

$$\dot{m} = \rho * \dot{V} = 1.1236 * 0.55331 = 0.6217 \text{ kg/s}$$

Velocity in branches?

$$\dot{V}_2 = v A = 5.8 * \text{Pi} * 0.268^2 / 4 = 0.32718 \text{ m}^3/\text{s}$$



$$m = \rho \cdot v = 1.1250 \cdot 0.55331 = 0.6217 \text{ kg/s}$$

Velocity in branches?

$$\dot{V}_2 = v \cdot A = 5.8 \cdot \text{Pi} \cdot 0.268^2/4 = 0.32718 \text{ m}^3/\text{s} \quad - 2$$

$$\dot{V}_{3+4} = V_1 - V_2 = 0.55331 - 0.32718 = \underline{0.22613} \text{ m}^3/\text{s}$$

$$\dot{V}_3 = 0.22613 / 2 = \underline{0.11307} \text{ m}^3/\text{s}$$

$$v = \dot{V}_3/A_3 = 0.11307 / (\text{Pi} \cdot 0.142^2/4) = 7.13971 \text{ m/s}$$



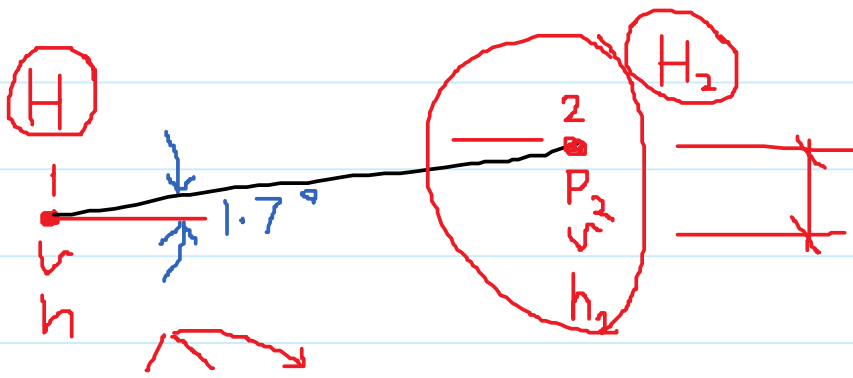
Head (p240)



$$P = \rho g h$$

$$h = \frac{P}{\rho g}$$

Q14: A 539mm diameter pipeline carries water at 4.8 m/s up a hill sloping at 1.7 degs. After a distance of 178 m the pressure is 54 kPa. (a) What is the total head?



$$h = \frac{P}{\rho g}$$

$$h_v = \frac{v^2}{2g}$$

$$H = h_p + h_v + h$$

$$H_1 = H_2$$

Find H at point 2

(since we know the pressure there)

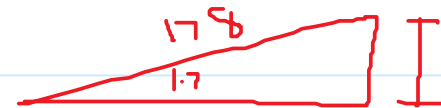
$$H_2 = h_{p2} + h_{v2} + h_2$$

$$h_{p2} = \frac{54000}{(1000 * 9.81)} = 5.5046 \text{ m} \leftarrow$$

$$h_{v2} = \frac{4.8^2}{(2 * 9.81)} = 1.1743 \text{ m} \leftarrow$$

$$h = 178 * \sin(1.7) = 5.2806 \text{ m} \leftarrow$$

$$H_2 = 5.5046 + 1.1743 + 5.2806 = \underline{11.9595 \text{ m}} \leftarrow$$



Bernoulli

BERNOULLI EXAMPLES

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + h_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + h_2$$

NO FRICTION
NO HEAT LOSS

Same Height

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + h_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + h_2$$

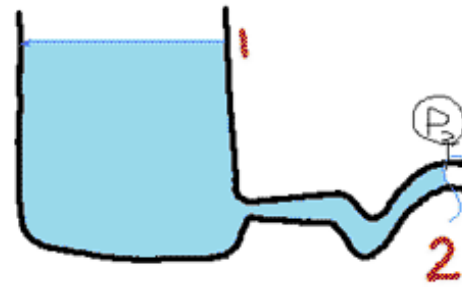
CANCEL



Surface at Atmosphere (Gauge p=0)

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + h_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + h_2$$

If large tank
velocity = 0



Now add nozzle to atmosphere...

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + h_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + h_2$$

atm large atm

$$\frac{v_2^2}{2g} = h_1 - h_2 = \Delta h$$

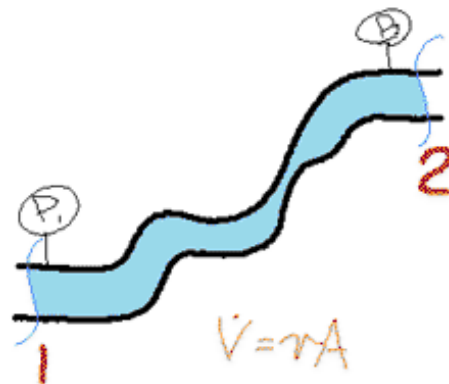
$$v = \sqrt{2g\Delta h}$$



Same diameter (or parallel pipe)

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + h_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + h_2$$

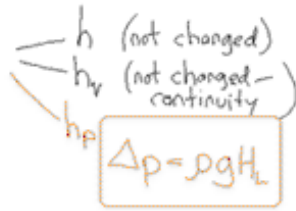
EQUAL



$$V = vA$$

HEAD LOSS $\begin{cases} h & \text{(not changed)} \\ h_f & \text{(not changed)} \end{cases}$

HEAD LOSS



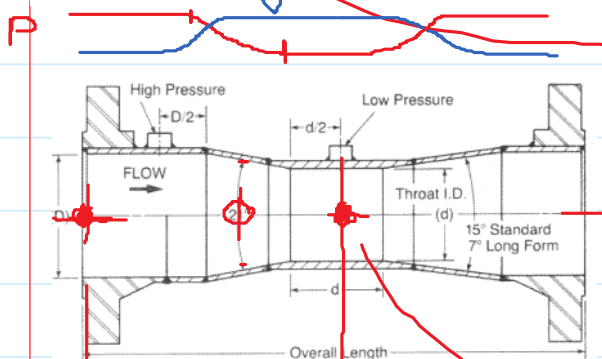
$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + h_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + h_2 + H_L$$

Head loss

Bernoulli Eg (p241)

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + h_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + h_2 + H_L$$

Q16: A horizontal venturi in a pipe of ID = 81 mm has a throat ID = 54 mm. Upstream the water pressure is 52 kPa and velocity is 4.2 m/s. (a) Find water velocity in the throat.



① $P_1 = 52 \text{ kPa}$
 ② $P_2 ?$
 VACUUM

$$V_1 = V_2$$

$$v_1 A_1 = v_2 A_2$$

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + \cancel{h_1} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + \cancel{h_2} + H_L$$

Continuity:

$$V_1 = vA = 4.2 * (\pi * 0.081^2 / 4)$$

$$= 0.021643 \text{ m}^3/\text{s}$$

$$v_2 = V / A = 0.021643 / (\pi * 0.054^2 / 4)$$

$$= \underline{9.45018 \text{ m/s}}$$

Find Pressure at 2

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + \cancel{h_1} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + \cancel{h_2} + \cancel{H_L}$$

$h_{p1} \quad h_{v1} \quad h_{v2}$

$$- h_{p1} = 52000 / 9810 = 5.3007 \text{ m}$$

$$h_{v1} = 4.2^2 / (2 * 9.81) = 0.8991 \text{ m}$$

$h_1 = \text{ignore}$

$h_{p2} = ?$

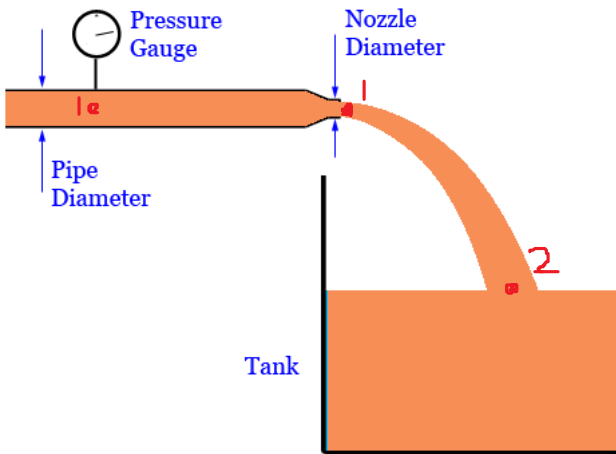
$$h_{v2} = 9.45018^2 / (2 * 9.81) = 4.55178 \text{ m}$$

$h_2 = \text{ignore}$

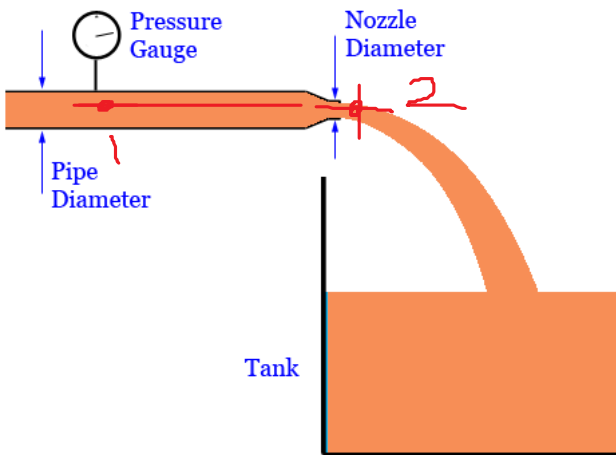
$$p_2 / \rho g = 5.3007 + 0.8991 - 4.55178 = \underline{1.64802 \text{ m}}$$

$$p_2 = 1.64802 * 9810 = \underline{16167 \text{ Pa}} = \underline{16.167 \text{ kPa}}$$

Q18: A 164mm diam pipe discharges 70 kg/s oil (RD=0.85) through a nozzle into a tank at atmospheric pressure. Pressure in the pipe is 128 kPa. (a) Find velocity of oil through the nozzle.



$$\cancel{\frac{p_1}{\rho g}} + \frac{v_1^2}{2g} + h_1 = \cancel{\frac{p_2}{\rho g}} + \frac{v_2^2}{2g} + h_2 + \cancel{H_L}$$



Q18: A d_1 164mm diam pipe discharges v_2 70 kg/s oil (RD=0.85) through a nozzle into a tank at atmospheric pressure. Pressure in the pipe is P_1 128 kPa. (a) Find velocity of oil through the nozzle.

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + \cancel{h_1} = \cancel{\frac{p_2}{\rho g}} + \frac{v_2^2}{2g} + \cancel{h_2} + \cancel{H_L}$$

v_2

$$\dot{V} = vA$$

Area ?

Find v_1 from $m = \rho vA$

$$v_1 = \frac{\dot{m}}{\rho A} = 70 / (850 * \pi * 0.164^2 / 4) = 3.8985 \text{ m/s}$$

$$h_{p1} = 128000 / (9.81 * 850) = 15.3505 \text{ m}$$

$$h_{v1} = 3.8985^2 / (2 * 9.81) = 0.7746 \text{ m}$$

h_1 - ignore

Note:
Kinckyn 230 (h)

$$h_{v1} = 3.8985^2 / (2 * 9.81) = 0.7746 \text{ m}$$

$$h_1 = \text{ignore}$$

$$h_{p2} = 0 \text{ (atmosphere)}$$

$$h_{v2} = ?$$

$$h_2 = \text{ignore}$$

$$\text{From Bernoulli;}$$

$$v_2^2 / 2g = h_{p1} + h_{v1}$$

$$= 15.3505 + 0.7746 = 16.1251 \text{ m}$$

$$v_2^2 = 16.1251 * 2 * 9.81 = 316.3745 \text{ m}$$

$$v_2 = 316.3745^{0.5} = 17.7869 \text{ m/s}$$

Note:

Kinsky p 239 (b)

A2 = A(0.02)

This means Area (0.02 diameter)

Q19: Find nozzle diameter;

$$\dot{V}_1 = \dot{V}_2$$

$$v_1 A_1 = v_2 A_2$$

$$A_2 = v_1 A_1 / v_2$$

$$= 3.8985 * (\pi * 0.164^2 / 4) / 17.7869$$

$$= 0.00462993 \text{ m}^2$$

$$\text{From } A = \pi R^2 \text{ then } R = (A / \pi)^{0.5}$$

$$\text{Radius} = (0.00462993 / \pi)^{0.5} = 0.0383895 \text{ m}$$

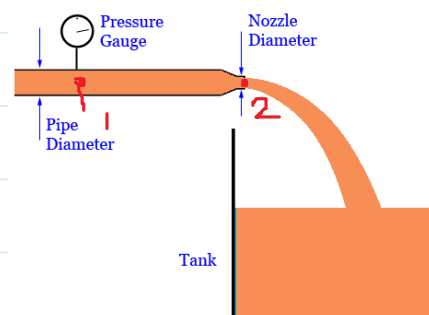
$$\text{So Diam} = 2 * 0.0383895 = 0.076779 \text{ m } 76.78 \text{ mm}$$

JUST FOR FUN...

What happens if we change nozzle diameter to 100mm?

Q18: A 164mm diam pipe discharges 70 kg/s oil (RD=0.85) through a nozzle into a tank at atmospheric pressure. Pressure in the pipe is 128 kPa. (a) Find velocity of oil through the nozzle.

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + \cancel{h_1} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + \cancel{h_2} + \cancel{H_L}$$



$$v_1 = 3.8985 \text{ m/s}$$

$$h_{v1} = 3.8985^2 / (2 * 9.81) = 0.7746 \text{ m}$$

$$\text{Continuity... } v_1 A_1 = v_2 A_2$$

$$\dot{V} = v_1 A_1 = 3.8985 * \pi * 0.164^2 / 4 = 0.082352 \text{ m}^3/\text{s}$$

$$A_2 = \pi * 0.100^2 / 4 = 0.0078539 \text{ m}^2$$

$$v_2 = \dot{V} / A_2 = 0.082352 / 0.0078539 = 10.4854913 \text{ m/s}$$

$$h_{v2} = 10.4854913^2 / (2 * 9.81) = 5.6037476 \text{ m}$$

$$\text{From Bernoulli;}$$

$$h_{p1} + h_{v1} = h_{v2}$$

$$h_{p1} = 5.6037476 - 0.7746 = 4.8291476 \text{ m}$$

$$p_1 / \rho g = 4.8291476$$

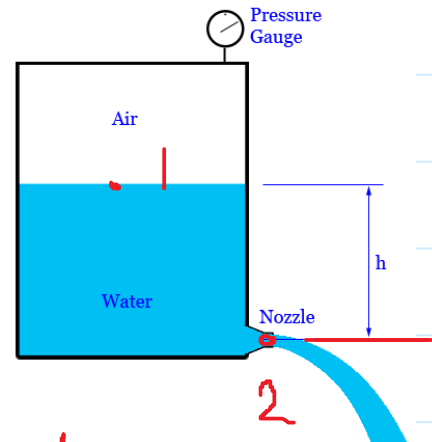
$$p_1 = 4.8291476 * \rho g$$

$$= 4.8291476 * 850 * 9.81$$

$$= 40267.8 \text{ Pa } (40 \text{ kPa})$$

Bernoulli Eg2

Q19: A water tank is filled to $h = 3.3$ m above a nozzle discharging to atmospheric pressure. If the pressure gauge reads 69 kPa, find the velocity of water at the nozzle.



$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + h_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + h_2 + H_L$$

Handwritten annotations: '7' under $\frac{p_1}{\rho g}$, '3.3' under h_1 , and 'atm' with a bracket under $\frac{p_2}{\rho g}$. Red arrows point from the text above to the corresponding terms in the equation.

Compare points 1 & 2:

Look for surfaces, exit to atmosphere (nozzles), measured point (such as pressure gauge). Always use centreline of pipes, nozzles. Gases considered constant pressure (no height effect so ignore $P = \rho gh$)

$$h_{p1} = 69000 / (9.81 * 1000) = 7.0336 \text{ m}$$

$$h_{p1} = 69000 / (9.81 * 1.2) = 5.8614 \text{ km (air!!!)}$$

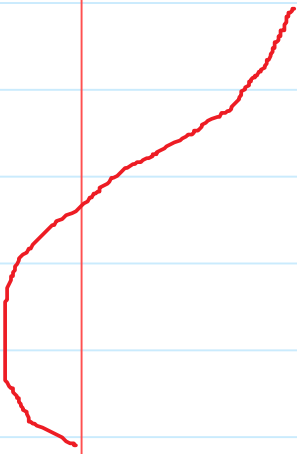
$$h_{v1} = (\text{ignore due to large diam tank})$$

$$h_1 = 3.3 \text{ m}$$

$$h_{p2} = (\text{ignore due to atmospheric pressure})$$

$$h_{v2} = 7.0336 + 3.3 = 10.3336 \text{ m}$$

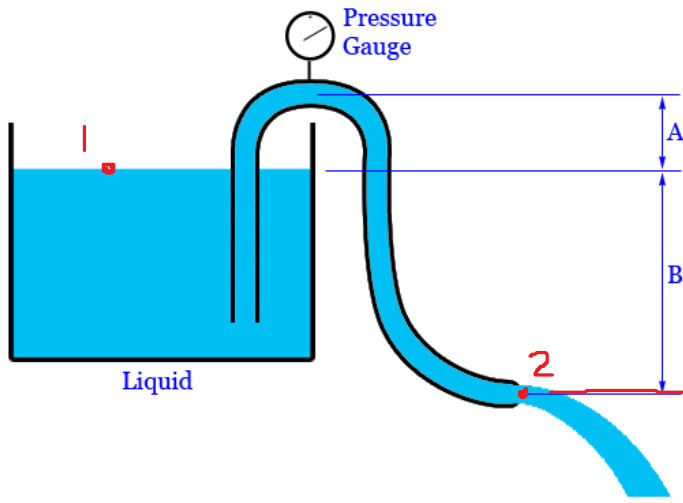
$$v_2 = (10.3336 * (2 * 9.81))^{0.5} = 14.2389 \text{ m/s}$$



Bernoulli Eg3

Thursday, 18 August 2011
12:10 PM

Q21: A siphon drains petrol (RD 0.737) from a tank. Tube diam=14mm, nozzle diam=9.6mm. Dim A=1.6m and B=1.4m. (a) How long would it take to siphon 27 litres?



- $h_{p1} = 0 \text{ m}$
- $h_{v1} = 0 \text{ m}$
- $h_1 = 1.4 \text{ m}$
- $h_{p2} = 0 \text{ m}$
- $h_{v2} = ?$
- $h_2 = 0 \text{ m}$

$$\cancel{\frac{p_1}{\rho g}} + \cancel{\frac{v_1^2}{2g}} + h_1 = \cancel{\frac{p_2}{\rho g}} + \frac{v_2^2}{2g} + \cancel{h_2} + H_L$$

$$\cancel{\frac{p_1}{\rho g}} + \cancel{\frac{v_1^2}{2g}} + h_1 = \cancel{\frac{p_2}{\rho g}} + \frac{v_2^2}{2g} + \cancel{h_2} + H_L$$

$$h_1 = 1.4 = v_2^2 / 2g$$

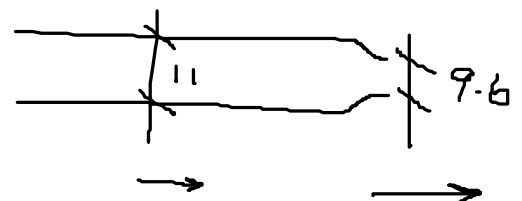
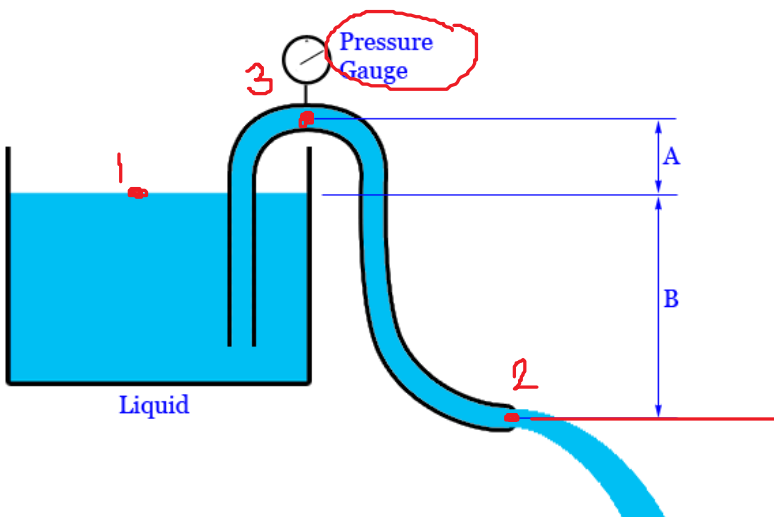
$$v_2 = (1.4 * 2 * 9.81)^{0.5} = 5.241 \text{ m/s}$$

$$\dot{V} = vA = 5.241 * \text{Pi} * 0.0096^2 / 4 = 0.000379356 \text{ m}^3/\text{s}$$

$$= 0.000379356 * 1000 = 0.379356 \text{ L/s}$$

$$\text{Time} = 27 / 0.379356 = 71.173252 \text{ s}$$

Q22: (cont) A siphon drains petrol (RD 0.737) from a tank. Tube diam=14mm, nozzle diam=9.6mm. Dim A=1.6m and B=1.4m. (b) What is the pressure reading? (at the top of the tube)



$$v_3 = v_2$$

$$v_3 A_3 = v_2 A_2$$

Very common to use continuity before we can get

And some more...

Tuesday, 23 August 2011
6:23 PM

Pasted from

PITOT TUBE

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + h_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + h_2$$

$P_1 = P_2 + P_{vel}$ (pressure)
 $H_T = h_p + h_v$ (head)

$p_2 = \rho v_1^2 / 2$
 $v_1 = (2p_2 / \rho)^{0.5}$

Venturi

Area decrease
 → Velocity increase
 → Pressure drops

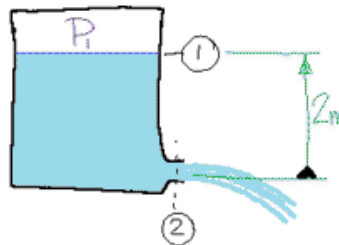
$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + h_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + h_2$$

Ex 11.3 p244

$P_1 = 50 \text{ kPa}$

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + h_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + h_2$$

big *atm* *datum*



$$\frac{50 \times 10^3}{1000 \times 9.81} + 2 = \frac{v_2^2}{2 \times 9.81}$$

$$50968 + 2 = \frac{v_2^2}{2 \times 9.81}$$

$$139.24 = v_2^2 \rightarrow v_2 = 11.8 \text{ m/s}$$

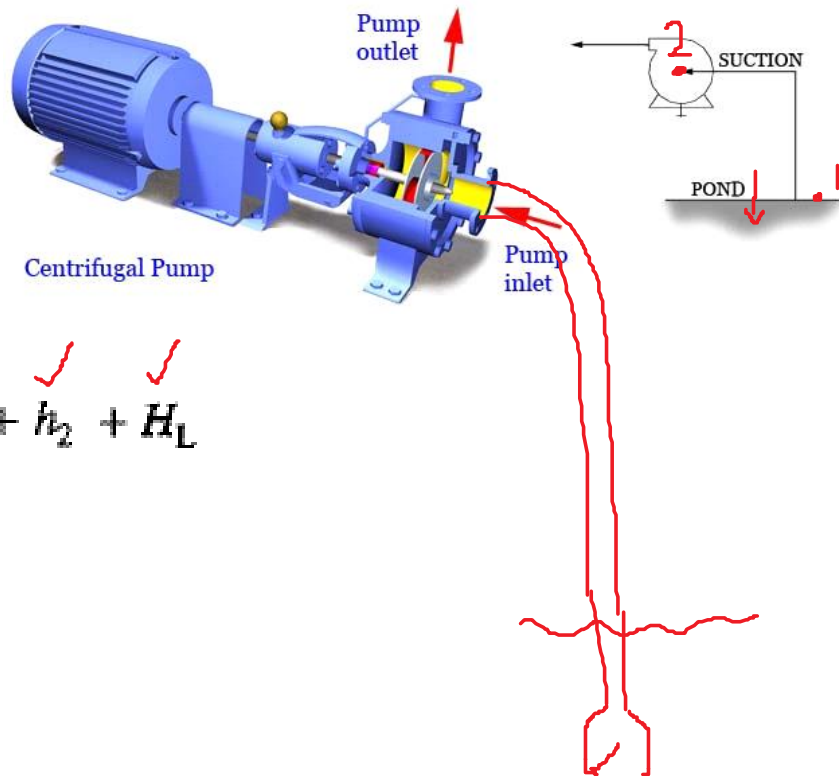
Head Loss

Tuesday, 6 August 2013
6:58 PM

Q20: Salt water (RD=1.04) is pumped at 28.1 L/s through a pipe (ID=80 mm) from a pond 2.7 m below the pump. Head loss in suction line is 1.6 m. Find gauge pressure at pump inlet.

$$H_L = 1.6\text{m}$$

$$h_2 = 2.7\text{m}$$



$$\cancel{\frac{p_1}{\rho g}} + \cancel{\frac{v_1^2}{2g}} + \cancel{h_1} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + h_2 + H_L$$

$$V = vA$$

$$v_2 = V/A$$

Duct

Tuesday, 23 August 2011
6:33 PM

Q23: Air in a 327mm square inlet flows at 14 m/s where a water manometer $X_1=214\text{ mm}$. The duct reduces to 195mm tall at the outlet 13m below. (Air Spec Vol = $0.85\text{m}^3/\text{kg}$). (a) Find mass flow rate.

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + h_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + h_2 + H_L$$

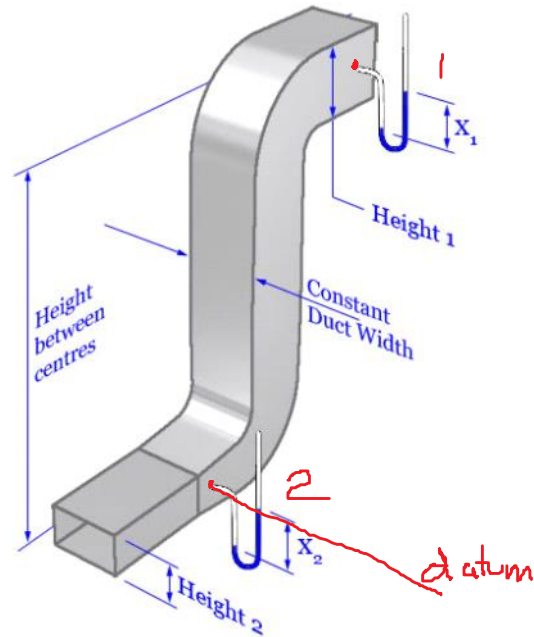
Mass flow rate... (kg/s)

$$\dot{V} = vA$$

$$\dot{m} = \rho \dot{V} = \rho vA$$

$$\text{Density} = 1/v = 1/0.85 = 1.1765\text{ kg/m}^3$$

$$\dot{m} = 1.1765 * 14 * (0.327 * 0.327) = 1.7612\text{ kg/s}$$



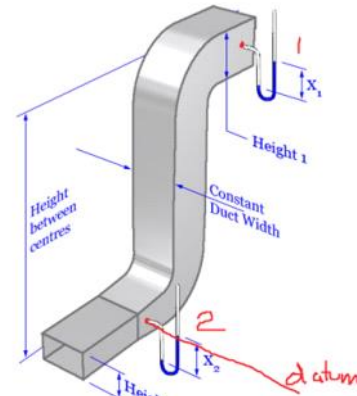
Q24: (cont) Air in a 327mm square inlet flows at 14 m/s where a water manometer $X_1=214\text{ mm}$. The duct reduces to 195mm tall at the outlet 13m below. (Air Spec Vol = $0.85\text{m}^3/\text{kg}$). (b) Find velocity at 2

$$\text{Continuity: } v_1 A_1 = v_2 A_2$$

$$v_2 = v_1 A_1 / A_2$$

$$= 14 * 0.327^2 / (0.195^2)$$

$$= 23.4769\text{ m/s}$$



Q25: (cont) Air in a 327mm square inlet flows at 14 m/s where a water manometer $X_1=214\text{ mm}$. The duct reduces to 195mm tall at the outlet 13m below. (Air Spec Vol = $0.85\text{m}^3/\text{kg}$). (c) Manometer reading at 2

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + h_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + h_2 + H_L$$

Handy Hints:

Manometer: $P = \rho gh$

"Specific" = per kg

" " = per second

h_p (m)
working fluid is AIR!

Manometer: (water)

$$P_1 = \rho gh = 1000 * 9.81 * 0.214 = 2099.34\text{ Pa}$$

Air at point 1...

$$h_p = 2099.34 / (1.1765 * 9.81) = 181.8955\text{ m}$$

$$h_v = 14^2 / (2 * 9.81) = 9.9898\text{ m}$$

$$h = 13\text{ m}$$

$$\text{Total Head } H_1 = 181.8955 + 9.9898 + 13 = 204.8853\text{ m}$$

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + h_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + h_2 + H_L$$

$$h_{v2} = (23.4769^2) / (2 * 9.81) = 28.092\text{ m}$$

Then re-arrange Bernoulli eqn...

$$h_{p2} = 181.8955 + 9.9898 + 13 - 28.092 = 176.7933\text{ m}$$

Convert pressure head to pressure...

$$p_2 = h_{p2} \cdot \rho g = 176.7933 \cdot (1.1765 \cdot 9.81) = 2,040.5 \text{ Pa}$$

Now convert to manometer reading...

$$P_1 = \rho g h = 1000 \cdot 9.81 \cdot ??? = 2040.5 \text{ Pa}$$

...

At point 1...

$$h_p = 2099.34 / (1.1765 \cdot 9.81) = 181.8955 \text{ m}$$

$$h_v = 14^2 / (2 \cdot 9.81) = 9.9898 \text{ m}$$

$$h = 13 \text{ m}$$

$$\begin{aligned} \text{Total Head } H_1 &= 181.8955 + 9.9898 + 13 \\ &= 204.8853 \text{ m} \end{aligned}$$

$$11\% \text{ of } H = 204.8853 \cdot 0.11 = 22.5374 \text{ m}$$

Head loss of 11% means 11% of the TOTAL HEAD. and Bernoulli's eqn (as *distinct from Bernanke's eqn*) must balance on both sides...

$$H_1 = H_2.$$

$$\begin{aligned} H &= h_p + h_v + h \\ &= p/\rho g + v^2/2g + h \end{aligned}$$

Q27: (cont) Air in a 327mm square inlet flows at 14 m/s where a water manometer X1=214 mm. The duct reduces to 195mm tall at the outlet 13m below. (Air SV=0.85m³/kg). (e) Find manometer 2 with head loss of 11%

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + h_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + \cancel{h_2} + H_L$$

H_1 ? 28.092 m 22.5374 m
 204.8853