3. The limiting value of static friction is given by

$$F_f = \mu F_n$$

4. The value of coefficient of static friction (μ) depends on the nature and condition of the surfaces in contact, but is independent of the areas in contact.
5. In general, kinetic friction is less than the limiting static friction.

Example 7.1

A body of mass 5 kg rests on a horizontal surface and the coefficient of friction between the two surfaces is 0.33. What horizontal force will be required to start the body moving?

Solution (Refer to Fig. 7.1)

Weight of body: $F_w = m.g = 5\text{ kg} \times 9.81\frac{\text{N}}{\text{kg}} = 49.05\text{ N}$
Normal force: $F_n = F_w = 49.05\text{ N}$
Limiting friction: $F_f = \mu F_n = 0.33 \times 49.05\text{ N} = 16.2\text{ N}$
Force required to just start the body moving is $F_p = F_f = 16.2\text{ N}$.

Example 7.2

In an experiment to determine the coefficient of static friction, a horizontal force of 50 N was required to start a 10 kg block moving on a horizontal surface. What was the value of the coefficient?

Solution (Refer to Fig. 7.1)

$$F_n = F_w = \text{m.g} = 10\text{ kg} \times 9.81\frac{\text{N}}{\text{kg}} = 98.1\text{ N}$$
$$F_f = F_p = 50\text{ N}$$
$$\mu = \frac{F_f}{F_n} = \frac{50\text{ N}}{98.1\text{ N}} = 0.51$$

Example 7.3

A 100 kg block rests on a plate as shown in Figure 7.2. The coefficient of friction between all surfaces is 0.2. Determine the force required to pull the plate from under the block.

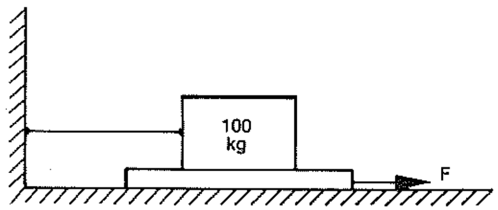

Fig. 7.2

Solution

Normal force:

$$F_n = F_w = \text{m.g} = 100\text{ kg} \times 9.81\frac{\text{N}}{\text{kg}} = 981\text{ N}$$

Friction force:

$$F_f = \mu F_n = 0.2 \times 981\text{ N} = 196.2\text{ N}$$

The applied force F_p must overcome friction between two pairs of surfaces in contact.

Therefore, $F_p = 2F_f = 2 \times 196.2\text{ N} = 392.4\text{ N}$

Problems

7.1 A block of mass 2 kg rests on a horizontal table and the coefficient of static friction between the surfaces is 0.28. What horizontal force will be required to start the block moving?

7.2 A block of wood having a mass of 5 kg rests on a horizontal table. A horizontal force of 12 N is just sufficient to cause it to slide. What is the coefficient of static friction between the surfaces?

7.3 A 2 tonne girder is pulled along a horizontal floor by a winch. The coefficient of friction between the girder and the floor is 0.35. What is the tension in the horizontal rope between the girder and the winch?

7.4 A 20 kg block rests on a horizontal surface and is attached to a mass of 3.5 kg by a cable as shown in Figure 7.3. The pulley is frictionless and the coefficient of static friction is 0.2.

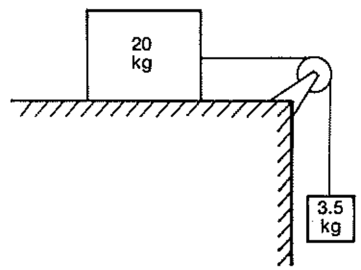

Fig. 7.3

Prove that the mass is not sufficient to start the block moving. What is the additional mass that would be required to start motion?

7.5 Each of the two blocks in Figure 7.4 has a mass of 10 kg and the coefficient of friction between all surfaces is 0.3. Determine the force F_p required to pull one block from under the other.

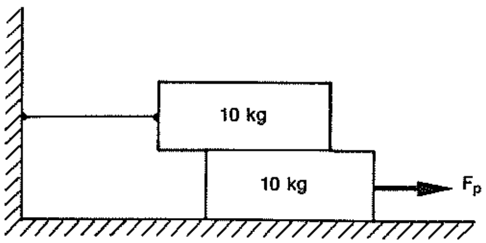

Fig. 7.4

7.6 In an automatic materials handling operation metal blocks 1.5 kg each are pushed one at a time from the bottom of a stack six blocks high as shown in Figure 7.5. If the coefficient of friction is 0.25, determine the horizontal force required.

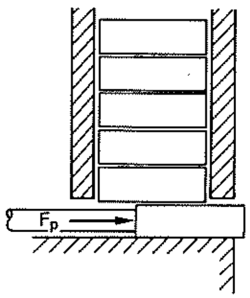

Fig. 7.5

7.7 If the steel straps shown in Figure 7.6 rely on friction to transmit a load of 3 kN, what should the force be in each of the four bolts? Take $\mu = 0.2$.

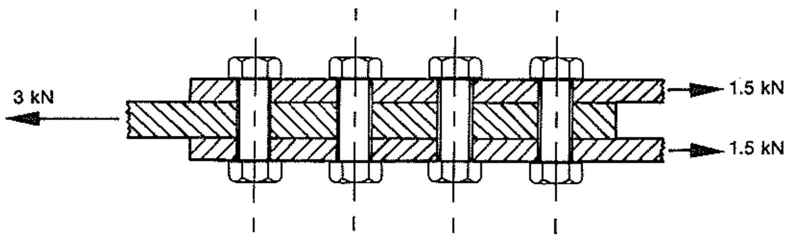

Fig. 7.6

7.8 What is the braking torque applied to the brake drum of 300 mm diameter (Fig. 7.7) if the brake shoes are pressed to the drum with a force of 800 N each? The coefficient of friction is 0.6.

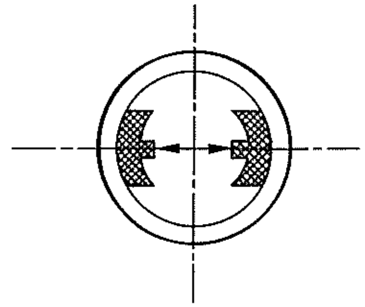

Fig. 7.7

7.9 In the brake shown in Figure 7.8, the coefficient of friction between the brake shoe and the drum is 0.45. Find the smallest value of force F required to prevent rotation of the drum against an applied torque of 75 N.m.

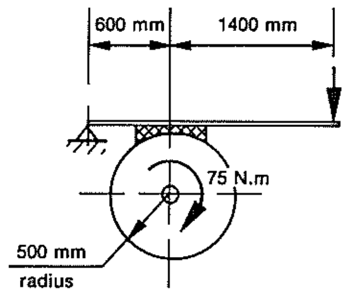

Fig. 7.8

7.10 A 35 kg cabinet, 0.75 m × 0.75 m × 1.8 m high, is pushed by a gradually increasing horizontal force applied 1.5 m above floor level. If the coefficient of friction between the cabinet and the floor is 0.4, determine if the cabinet will slide or tip.

7.3 The angle of friction

Let us now consider another approach to the analysis of friction on a horizontal plane. Referring to Figure 7.9, it is possible to combine the force of friction (F_f) and the normal reaction (F_n) into a single resultant force (F_r), representing a total reaction at the surface of contact to the action of weight (F_w) and applied force (F_p).

Remembering that $\mu = \frac{F_f}{F_n}$, we write

$$\mu = \frac{F_f}{F_n} = \frac{F_w \sin \theta}{F_w \cos \theta} = \tan \theta$$

$$\text{but } \mu = \tan \phi$$

$$\text{therefore } \tan \theta = \tan \phi$$

$$\text{or } \theta = \phi$$

The value of the angle of inclination corresponding to impending motion is called the **angle of repose.** Clearly, the angle of repose is equal to the angle of static friction.

Understanding the concept of the angle of repose is important in the design of bins and hoppers for the storage and handling of granular materials, and for calculating the steepest angle to the horizontal which can be made by the inclined surface of a heap of loose material or an embankment.

Example 7.5

What is the steepest ramp on which a car can stand without slipping down, if the coefficient of friction between the tyres and the ramp surface is 0.8?

Solution

$$\begin{aligned} \text{Angle of repose} &= \text{Angle of friction } \phi \\ &= \tan^{-1} \mu \\ &= \tan^{-1} 0.8 \\ &= 38.7° \end{aligned}$$

Problems

7.11 Determine the angle of friction corresponding to a coefficient of static friction of 0.6.

7.12 If the normal and frictional forces between two surfaces which are about to slip are 100 N and 35 N respectively, determine the coefficient of static friction, the angle of friction and the magnitude of the resultant force between the surfaces.

7.13 If the normal reaction between two surfaces is 120 N and the coefficient of friction is 0.25, determine the magnitude and direction of the total reaction for the case of limiting friction.

7.14 A body of 2 kg mass rests on a board which is gradually tilted until, at an angle of 27° to the horizontal, the body begins to move down the plane. Determine the coefficient of friction and the magnitude of the normal and frictional forces when the body begins to slip.

7.15 Solve problem 7.1 graphically.

7.16 Solve problem 7.2 graphically.

7.17 Solve problem 7.3 graphically.

7.18 Solve problem 7.4 graphically.

7.19 Solve problem 7.5 graphically.

7.20 Solve problem 7.6 graphically.

7.5 The inclined plane

The inclined plane is one of the earliest mechanical devices used by people in their efforts to influence their environment. It was used as a simple machine for moving and lifting heavy stones for the construction of buildings and pyramids in the ancient world. Today it is still used as an element of modern machinery in the form of the power screw. Many other mechanical devices, such as cams, wedges and knife-like metal cutting tools, also operate on the inclined plane principle. Friction is a prominent and unavoidable feature of the operation of the inclined plane.

Problems involving friction on an inclined plane can be solved mathematically or graphically. The mathematical solution is best suited for problems in which the applied force is acting parallel to the plane, while the graphical method is useful for solving forces inclined to the plane at any angle. To some extent the choice of a method is a matter of personal preference.

Regardless of the method used, it is very important to remember that friction always opposes motion and to show all active forces accordingly.

Example 7.6

A 200 kg block rests on a 25° incline as shown in Figure 7.12(a). The coefficient of friction between the block and the plane is 0.2. Determine the magnitude of force F_p acting along the inclined plane required.

(a) to start the block moving up the inclined plane, and

(b) to prevent it slipping down the inclined plane.

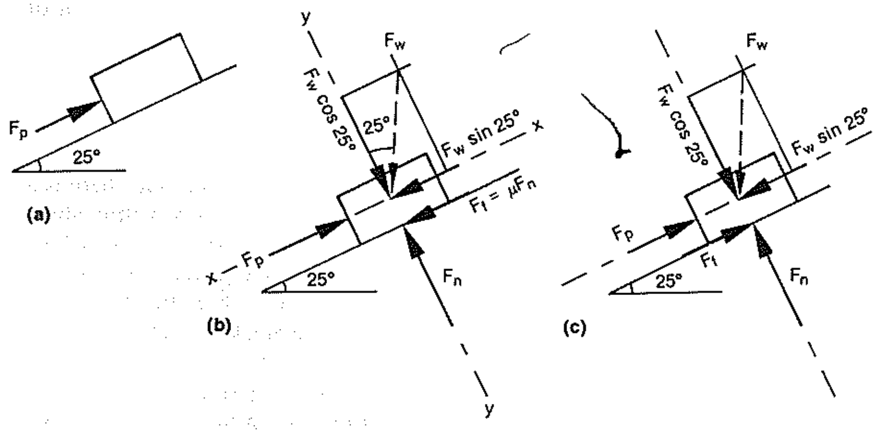

Fig. 7.12

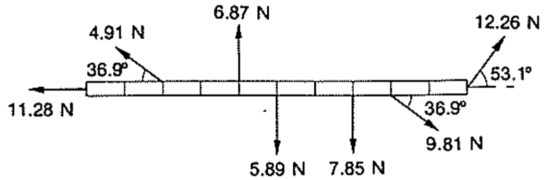

Fig. 1

3.12 (a) 9.81 mN
(b) 9.81 N
(c) 9.81 kN
(d) 39.2 N
(e) 0.49 N
3.13 22.6 kN
3.14 (a) 98.1 N
(b) 98.1 N
(c) 98.1 N
3.15 3.43 kN and 12.8 kN
3.16 12.8 kN at 72.4° to horizontal
3.17 Fig. 1 (Answer)

4.14 123 N and 506 N ∠76°
4.15 3.61 kN ∠56.3° and 3 kN ↓
4.16 871 N ∠73.3° and 250 N ←
4.17 649 N ∠30° and 864 N ∠49.5°
4.18 4.19 kN and 5.44 kN ∠50.4°
4.19 59.5 N ∠33.7°
4.20 19.2 kN ∠66.8° 0.8 m

Chapter 4

4.1 13.2 N and 10.8 N
4.2 3.68 kN
4.3 46.0 kN 37.5 kN
4.4 2.76 kN 4.51 kN
2.85 kN 4.64 kN
4.5 178 N ∠65° No
4.6 2.04 t 0 28.3 kN
4.7 24.0 kN 20.8 kN 20.8 kN 0
16.0 kN 8.0 kN 16.0 kN
8.0 kN 20.8 kN 8.0 kN
4.8 (a) 7.55 kN ∠19.8°
(b) 49 kN ∠52.1°
(c) 8.82 kN ∠74.5°
(d) 0
4.9 192 N 279 N ∠20.7°
4.10 (a) 4.19 kN and 8.89 kN, 64.7° ↗
(b) 16.8 kN and 15.4 kN, 10.2° ↗
(c) 16.8 kN and 19.7 kN, 39.5° ↗
(d) 25.2 kN and 24.5 kN, 21.5° ↗
4.11 (a) 9.55 kN ∠10.3° and 1.71 kN ↓
(b) 8.31 kN ∠22.8° and 3.21 kN ↓
(c) 6.61 kN ∠40.9° and 4.33 kN ↓
4.12 406 N ∠25° and 520 N ∠45°
4.13 384 N and 348 N ∠17°

Chapter 5

5.1 13.5 N.m, Perpendicular to lever arm.
5.2 75 N.m
5.3 (a) 3 kN.m ↻ and 3 kN.m ↺
(b) 1.5 kN.m ↻ and 4.5 kN.m ↺
(c) 3 kN.m ↻ and 3 kN.m ↺
(d) 3.5 kN.m ↻ and 2.5 kN.m ↺
(e) 2.6 kN.m ↻ and 2.6 kN.m ↺
5.4 17.3 kN.m ↻
5.5 28.0 kN.m ↻ and 1.32 kN.m ↺
5.6 188 N.m
5.7 33.3 N
5.8 45 N.m
5.9 2 kN
5.10 50 N
5.11 123 N
5.12 5.56 kN
5.13 28 N.m
5.14 $\Sigma M = 0$
5.15 $\Sigma M = 0$
5.16 110 N
5.17 2 kN and 0.5 kN.m ↻
5.18 15 mm
5.19 1.3 kN and 1.95 kN.m ↺
5.20 49.1 N and 31.9 N.m ↻

Chapter 6

6.1 (a) F_L = 6 kN ↑ F_R = 9 kN ↑
(b) F = 2 kN ↓ M = 3.2 kN.m ↻
(c) F_L = 3 kN ↑ F_R = 3 kN ↑
(d) F_L = 23 kN ↑ F_R = 29 kN ↑
(e) F = 9 kN ↓ M = 16 kN.m ↻
(f) F_L = 11 kN ↑ F_R = 3 kN ↑
(g) F = 3 kN ↓ M = 0
(h) F = 0 M = 12 kN.m ↺
(i) F_L = 21.6 kN ↑ F_R = 19.4 kN ↑
(j) F_L = 2 kN ↑ F_R = 2 kN ↑
6.2 (a) 48 kN ↓ 4.13 m
(b) 550 kN ↓ 4.09 m
(c) 6.75 kN ∠79.5° 3.81 m
(d) 4.53 kN ∠79.9° 3.88 m
(e) 3.08 kN ∠80.5° 3.68 m
6.3 848 N ∠79.8° 4.47 m up along the ladder
6.4 666 N ∠11.9° 73.2 mm
6.5 33.1 kN ∠65°
4.05 m (perpendicular distance)

Chapter 7

7.1 5.49 N
7.2 0.245
7.3 6.87 kN
7.4 0.5 kg
7.5 88.3 N
7.6 40.5 N
7.7 1.88 kN
7.8 144 N.m
7.9 100 N
7.10 Tip
7.11 31°
7.12 0.35 19.3° 106 N
7.13 124 N at 14° to normal
7.14 0.51 17.5 N 8.91 N
7.15 5.49 N
7.16 0.245
7.17 6.87 kN
7.18 0.5 kg
7.19 88.3 N
7.20 40.5 N
7.21 878 N
7.22 8.49 kN
7.23 37.6 N
7.24 34.6 N
7.25 234 N 340 N
7.26 225 kg
7.27 0.313
7.28 22.8 kg 7.2 kg
7.29 1.67 kg 0.6 kg

Chapter 8

8.1 6.96 m/s
8.2 85 km/h 23.6 m/s
8.3 75 km/h
8.4 2.22 m/s^2
8.5 450 m/s
8.6 5625 m
8.7 1.07 m/s^2 13 s
8.8 0.605 m/s^2 25.7 s
8.9 No 2.04 m/s
8.10 2 km 58.8 km/h
8.11 11 s
8.12 22 s 193.6 m
8.13 3.16 s 31 m/s
8.14 62.4 m 3.57 s
8.15 2.50 km 4.54 km 75.8 s
8.16 5 m/s^2
8.17 5 N
8.18 276 N
8.19 125 kN
8.20 420 N
8.21 126 s
8.22 145 kN
8.23 291 s 3.23 km
8.24 24.6 kN
8.25 9.81 kN 11.4 kN
8.41 kN 8.81 kN
8.26 32.3 kN
8.27 23.6 N 19.6 N 15.6 N
8.28 320 N 160 N
8.29 0.866 m
8.30 0.377 m/s^2 9.43 N
8.31 0.318 kg
8.32 11.2 kg
8.33 0.467 m/s^2
8.34 3.05 m/s^2
8.35 17.1 kg, 2.15 s

Chapter 9

9.1 Hours hand: 1.389×10^{-3} rpm, 1.454×10^{-4} rad/s
Minutes hand: 1.667×10^{-2} rpm, 1.745×10^{-3} rad/s
Seconds hand: 1 rpm, 0.1047 rad/s
9.2 20 s 3100 rad
9.3 40 rad/s^2 1308 rev
9.4 152 rad/s 1450 rpm 184 rev
9.5 0.349 rad 244 mm
9.6 0.429 rad/s 0.214 rad/s^2
9.7 25.1 m/s