STRAIN AND ELASTICITY

STRAIN (p323)

Strain is Proportion of stretch.
Strain = extension / original length
(-) = (mm) / (mm)
It has no units, and should be a SMALL number with metals!

$e = x / L$ (Ivanoff & TAFE)

or

$\varepsilon = e / L$ (standard symbols)

Example: Steel bar is 2.7m long, extends 1.27mm under load. Find strain.

$\varepsilon = e / L$

$= 1.27 / 2700$
$= 0.0005$

Why use mm?
We want MPa. and N/mm^2 = MPa

Formulas

Wednesday, 30 July 2014
4:34 PM

	Standard	Textbook	Units
Stress	$\sigma = F / A$	$f = F / A$	MPa
Strain	$\varepsilon = e / L_o$	$e = x / L_o$	(none)
Modulus	$E = \sigma / \varepsilon$	$E = f / e$	MPa

We will use the international standard symbols. (Occasionally referring to Ivanoff symbols here and there)

MODULUS OF ELASTICITY p324

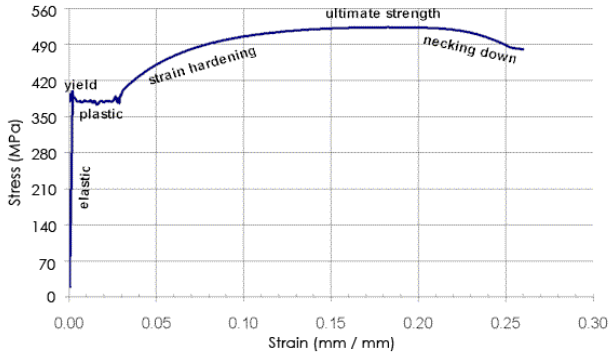
Stiffness = amount of stress to deform by certain proportion.

$E = f / e$ (Ivanoff & TAFE)

$E = \sigma / \varepsilon$ (standard symbols)

Other Names:

- Modulus of Elasticity
- Modulus of Stiffness
- Young's Modulus



The stiffness = slope of stress/strain curve.
But only in the ELASTIC region!
They are big numbers..
Steel: E = 200000 Mpa = 200 Gpa
We usually know the E if we know what it is made of.

$f = F / A$ $\quad e = x / L$ $\quad E = f / e$	IVANOFF
$\sigma = F / A$ $\quad \varepsilon = e / L$ $\quad E = \sigma / \varepsilon$	INTERNATIONAL

Q25.15: Steel Tape 5mm x 0.3mm, Force = 50N, Length = 40m. Stress? Elongation?

$\sigma = F / A$ $\quad \varepsilon = e / L$ $\quad E = \sigma / \varepsilon$

$\sigma = F / A = 50 / (5 * 0.3) = 33.3333$ Mpa

Now use σ in eqn 3: Find Strain

$\varepsilon = \sigma / E = 33.3333 / 200000 = 0.1667 \times 10^{-3} = 0.0001667$

Now use **e** in eqn 2: Find extension x

$x = \varepsilon * L = 0.0001667 * 40000 = 6.668$ mm

Do Q 25.17

Do Q 25:17
What is diameter of copper wire if force = 475N gives elongation 6mm over length 10m? (Answer 3mm)

$\sigma = F / A$ $\varepsilon = e / L$ $E = \sigma / \varepsilon$

$\varepsilon = x / L = 6 / 10000 = 0.0006$
Now use eqn 3
$E = \sigma / \varepsilon$, so $\sigma = E * \varepsilon = 112000 * 0.0006 = 67.2$ Mpa
$\sigma = F / A$, so $A = F / \sigma = 475 / 67.2 = 7.0685$ mm^2
Convert Area back to diameter...
$A = Pi * r^2$
$Pi * r^2 = 7.0685$
$r^2 = 7.0685 / pi = 2.25$
r = 2.25^0.5 = 1.5 mm
Diameter = 2 * 1.5 = 3 mm

Can go directly to diameter:
$A = Pi * (d/2)^2 = (Pi * (d)^2)/4$

Q3: A stainless steel (316) guitar string of diameter 0.3 mm and length 852 mm, is tensioned to 46.3 N. What is the stress?

$\sigma = F / A$ $\varepsilon = e / L$ $E = \sigma / \varepsilon$

Find Stress: $\sigma = F/A = 46.3/(Pi*0.15^2) = 655$ MPa

Get the stretch

$\sigma = F / A$ $\varepsilon = e / L$ $E = \sigma / \varepsilon$

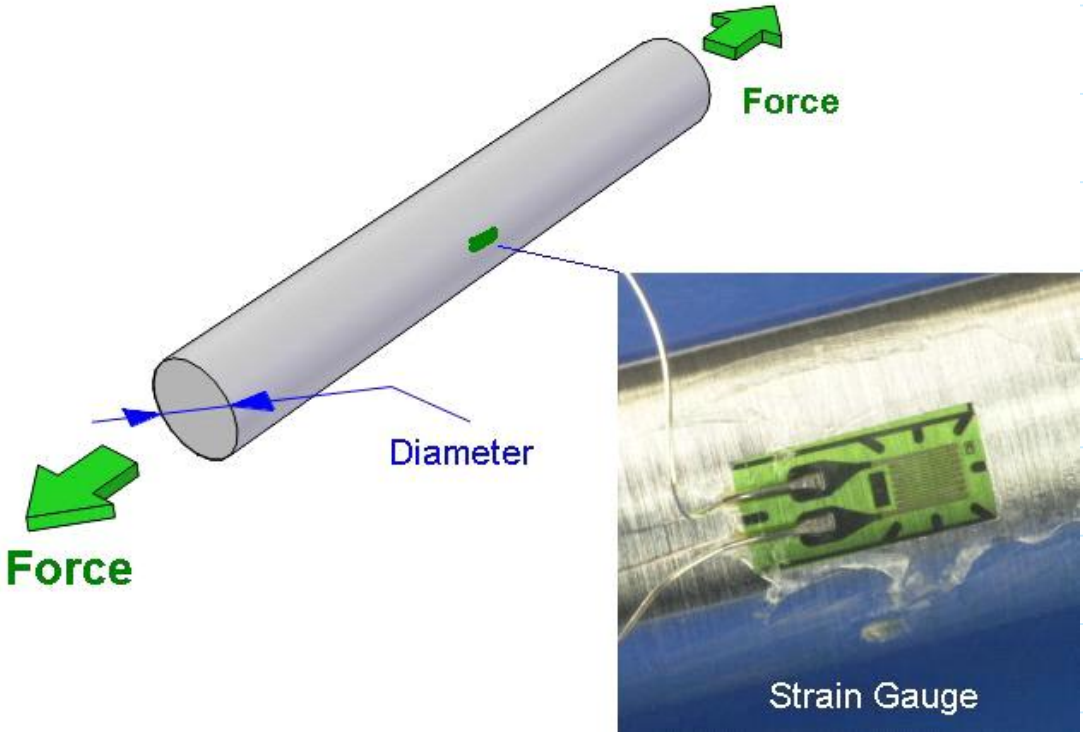
Find strain: $\varepsilon = \sigma / E = 655 / 195000 = 0.003359$

Find the stretch: $x = \varepsilon L = 0.003359 * 852 = 2.861868$ mm

Strain Gauges

$f = F / A$ $\quad e = x / L$ $\quad E = f / e$ (Ivanoff)

$\sigma = F / A$ $\quad \varepsilon = e / L$ $\quad E = \sigma / \varepsilon$ (International)



A strain gauge is a special film resistor stuck to the surface of the metal. When the metal stretches, the resistor stretches with it, and resistance changes. By measuring the resistance, we can determine the strain e.

Since we know the material, we know E. Then from $E = \sigma / e$ we can find stress! So $\sigma = eE$
Assume 0.1% strain in steel
Determine the stress…

$\sigma = F / A$ $\quad \varepsilon = e / L$ $\quad E = \sigma / \varepsilon$

$\sigma = E * e$
$= 200000*0.001$

σ = E * e
= 200000*0.001
= 200.0 MPa

Q9: This stainless steel (hardened 440) bar is 25.8 mm diameter. If the strain gauge reads 0.004254, what mass could it lift in tension?

E = σ/ε

σ = E*ε = 195000*0.004254 = 829.53 Mpa

A = Pi *(25.8/2)^2 = 522.792 mm2

σ = F/A

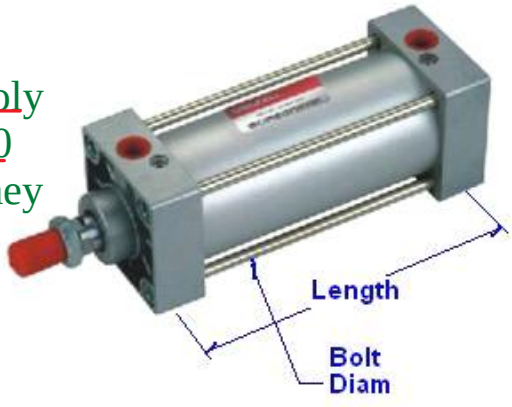
F = σA = 829.53*522.792 = 433,671.6478 N

Mass = 433,671.6478/9.81 = 44,207.0997 kg

Air Cylinder

Tuesday, 25 February 2014
8:27 PM

Q13: Four bolts (Grade 10.9, diam 5mm, Chrome-Moly 4130) clamp the cylinder ends with a total load of 950 kg. If the bolt length is 290 mm, by how much will they stretch?



$\sigma = F / A \qquad \varepsilon = e / L \qquad E = \sigma / \varepsilon$

Bolt Area = Pi*(5/2)^2 = 19.63495 mm^2
Force per bolt = (950/4)*9.81 = 2329.875 N
$\sigma = F/A = 2329.875/19.63495 = 118.65958$ Mpa
From E = f/e then
$\varepsilon = \sigma/E = 118.65958/200000 = 0.00059$
From e=x/l then
$x = \varepsilon L = 0.00059*290 = 0.1711$ mm

Thermal Expansion

$e = \alpha L_o \Delta T$

or from $\varepsilon = e / L$,

we get $e / L_o = \alpha \Delta T$

$\varepsilon = \alpha \Delta T$

ΔT = change in temperature

L_o = original length

α = coefficient of thermal expansion (mm/mmK)

Kelvin

The coefficient of thermal expansion is a property of a material (constant regardless of the size of the specimen)

Example p337: Aluminum rod, 1.2m long, 15°C. How long at 40°C?

α = 0.000024 mm/mmK

$\varepsilon = \alpha \Delta T$

0.024*25/1000 = 0.0006

Convert to extension x...

$\varepsilon = e / L$

e = εL = 0.0006 *1200 = 0.72 mm

Thermal stress is created when the specimen is prevented from expanding. To solve this, calculate the expansion, then compress the specimen back to the initial size. This will determine stress.

Example p337, Continued…

α = 0.000024 mm/mmK, e = **0.0006,** x = **0.72** mm

Now find the stress...

$\sigma = F / A$ $\varepsilon = e / L$ $E = f / \varepsilon$

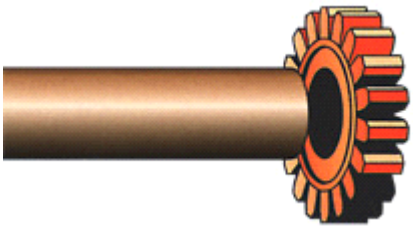
So for Aluminium, E = 70000Mpa, so the stress is;

σ = εE = 0.0006 *70000 = 42.0 MPa

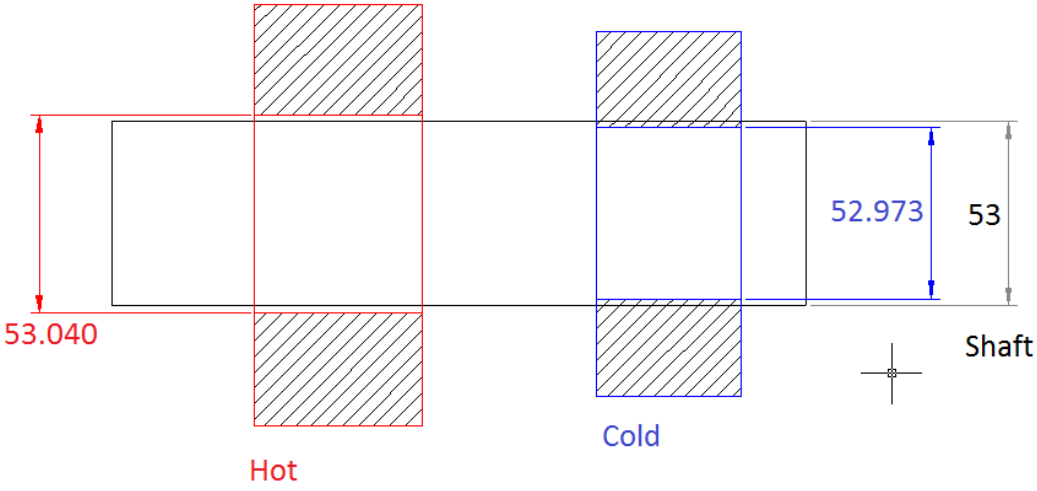
See shortcut on p340: $\sigma = E \alpha \Delta T$

Shrink Fit

Wednesday, 27 February 2013
10:50 AM



Q16: A shrink-fit steel (CS1020) gear is 27 microns smaller than the 53 mm diameter shaft. How much will the gear need to be heated to expand 40 microns beyond the shaft diameter?



$\varepsilon = \alpha \Delta T$

Find e first: $\varepsilon = e/L_o = (0.027+0.040)/(52.973) = 0.0013$

$\varepsilon = \alpha \Delta T$ so $\Delta T = \varepsilon/\alpha = 0.0013 / 0.000012 = 108.333333\ ^oC$