

Chapter 12: Helical Springs

There are many types of springs available to the engineer. They may be classified according to the type of load as shown below:

- Tension load:
 - helical cylindrical
 - flexible rod or bar
- Compression load:
 - helical cylindrical
 - helical spiral
 - multi-disc
 - flexible block
- Torsion load:
 - helical (cylindrical or spiral)
 - flexible bar, rod or block
 - flat spiral
- Bending load:
 - bar
 - flat leaf (single or multiple)

In theory, any elastic material can be used for a spring, but common materials used in mechanical engineering are:

- plain high carbon spring steel
- alloy steel (including stainless steel)
- spring brass, bronze or monel metal
- non-metals solids such as neoprene rubber
- gases such as air or nitrogen (gas springs)

For the purpose of this manual, helical cylindrical tension or compression springs made of round spring steel wire are the only type of spring treated.

As often occurs in mechanical design, springs may be custom designed and manufactured to engineers specification, or selected from the range of those produced by a spring manufacturer. It is generally more cost and time-effective to select an off-the-shelf spring rather than to design a custom spring. Also replacement springs are readily available if required. Therefore, the only time an engineer should contemplate custom-designing a spring is when the design requirements are such that no standard spring is available.

The range of stock helical compression and extension springs included in this manual have been taken from the latest A.S.P. (Automatic Springs Pty. Ltd.) catalogue available at the time of this publication (number 2/3). These springs are produced in hardened spring steel wire in sizes ranging from 0.355 mm to 5 mm in dia. Data on compression springs is given on pages 267 to 273 and on tension springs on pages 274 to 283.

Note : The catalogue number of these springs is based on imperial (inch) sizes. For example spring C0360-025-2000 refers to a compression spring, outside dia. 0.360 inches, wire dia. 0.025 inches, and free length 2.000 inches.

Wire diameter d

Standard wire diameters (in mm) used for springs are as listed in Table 1 below:

0.02	0.025	0.032	0.04	0.05	0.063	0.08	0.1	0.125	0.16	0.2	0.25	0.315	0.355
0.4	0.45	0.5	0.51	0.56	0.63	0.71	0.8	0.9	1.0	1.25	1.4	1.6	1.8
2.0	2.24	2.5	2.8	3.15	3.55	4.0	4.5	5.0	5.6	6.3	7.1	8.0	9.0
10.0	11.2	12.5	14.0	16.0									

Table 1 Standard wire diameters

Notes :

- Some of these sizes are not uniform because they are direct imperial (inch) size equivalents.
- The ASP range is for wire diameters in the range 0.355 - 5.0 mm only.

Spring diameter D

The diameter of a spring can be specified by the outside diameter, inside diameter or mean diameter. In the ASP catalogue, all springs are specified by the outside diameter. However, when designing a spring, the mean diameter is usually used.

Spring constant k

Because a spring deforms elastically, Hooke's Law applies and the change in length (or deflection) x is directly proportional to the change in force (or load) F . That is, the force-deflection diagram is a straight line as shown in Figure 1. This line passes through the origin because at zero load, there is zero deflection.

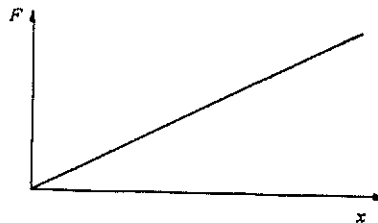


Figure 1 Force-deflection diagram for a spring

The slope of this line is known as the spring constant or spring rate. Using the symbol k :

$$k = \frac{F}{x} \quad \text{Formula 1}$$

Where F = total force (N) and x = total deflection (mm)

Or F = change in force (N) and x = change in deflection (mm)

Notes:

- In the ASP catalogue symbol R is used for the spring rate.
- Base units are N/m but conventionally deflection is expressed in mm so the units of k will be N/mm.

Pre-load

It is usually the case that a spring is pre-loaded, that is, it carries a certain load (or exerts a certain force) when the working deflection is zero. For example, consider the valve spring in a motor car engine. In the valve-closed position, the spring exerts a force to keep the valve closed. This is the pre-load force. As the valve opens, the spring deflects and the force increases to a maximum value when the maximum deflection is obtained in the fully-open position. This is shown in Figure 2.

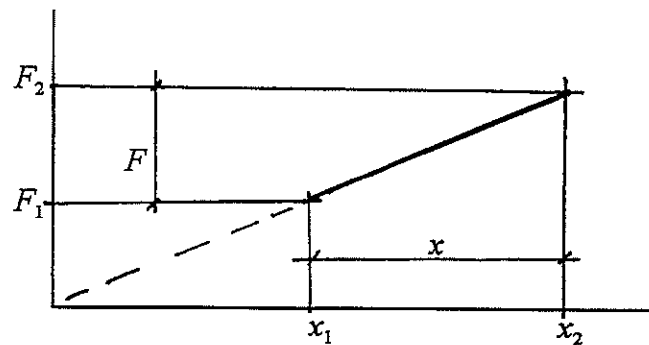


Figure 2 Force-deflection diagram with pre-load

In this figure :

- x_1 = pre-load deflection
- x_2 = total (maximum) deflection
- x = working deflection (change in deflection)
- F_1 = pre-load force
- F_2 = maximum force
- F = change in force

Example 1

A valve spring exerts a force of 200 N when the valve is closed and a force of 250 N when the valve is open. The working deflection is 8 mm.

Determine the spring constant, the pre-load deflection and the maximum deflection.

Solution

The change in force $F = 250 - 200 = 50 \text{ N}$

$$k = \frac{F}{x} = \frac{50}{8} = 6.25 \text{ N/mm}$$

The initial deflection $x_1 = \frac{200}{6.25} = 32 \text{ mm}$

The total deflection $x_2 = 32 + 8 = 40 \text{ mm}$ (or $\frac{200}{6.25}$)

Stock spring selection

Selection of tension or compression springs using the ASP catalogue is relatively straightforward and is illustrated in Example 2 which follows.

Example 2

A valve spring for a hydraulic pump is to have a lift of 10 mm and is to be closed by a compression spring. In the valve-closed position, the spring force is 250 N and in the valve-open position, the spring force is 350 N.

Select a stock ASP spring and complete the following table:

<i>Outside diameter</i> <i>mm</i>	<i>Wire diameter</i> <i>mm</i>	<i>Free length</i> <i>mm</i>	<i>Spring rate</i> <i>N/mm</i>	<i>Max deflection</i> <i>mm</i>

Solution

The spring constant $k = \frac{F}{x} = \frac{100}{10} = 10 \text{ N/mm}$

The maximum force is 350 N

Going to the ASP catalogue for compression springs (page 273) choose the closest spring: C1687-177-4000

The free length is 101.6 mm and the minimum recommended length $L_1 = 64.64 \text{ mm}$

Therefore the maximum deflection = $101.6 - 64.64 = 36.96 \text{ mm}$ (say 37 mm)

The table may now be completed:

<i>Outside diameter</i> <i>mm</i>	<i>Wire diameter</i> <i>mm</i>	<i>Free length</i> <i>mm</i>	<i>Spring rate</i> <i>N/mm</i>	<i>Max deflection</i> <i>mm</i>
42.85	4.5	101.6	9.77	37

Spring design

When an off-the-shelf spring is not available that will fulfil the design requirements, it is necessary to design a custom spring to the engineers specification. There are several methods available for spring design including nomograms and formula methods. Nomograms have a limited range but the formulas apply to any situation, so the formula method only will be treated here.

Spring index C

One of the important variables in the design of a spring is the spring index C which is defined as the ratio of the mean diameter to the wire diameter. That is :

$$C = \frac{D}{d} \quad \text{Formula 2}$$

Most springs used in engineering have a spring constant that lies in the range 4 to 15. As a rule-of-thumb, C increases as the size of the spring increases in accordance with the following table:

Size of spring	D mm	d mm	C
small	<8	<1	4-8
medium	8-24	1-4	8-12
large	>24	>4	12-15

Table 2 Typical values of the spring index C

It should be emphasised that this table is for *guidance only* for the purpose of initial trial. Springs often have C values that lie outside the range of values given in this table.

Allowable stress f_{all}

The maximum allowable stress in the spring under design or maximum load conditions depends upon the following factors:

- wire material properties
- diameter of the wire (small diameter wire has a higher allowable stress than large diameter wire)
- service conditions or duty which depends primarily upon the number of cycles required in the life of the spring and the amount of shock associated with the load. For convenience, three service conditions can be identified as listed in the following table:

<i>Duty</i>	<i>Number of cycles</i>	<i>Type of load</i>
Light	$<10^4$	static or gradually applied
Average (medium)	$10^4 - 10^6$	gradually applied - light shock
Heavy	$>10^6$	light-heavy shock

Table 3 Spring service conditions

For hard-drawn spring steel SAE 1065 grade, the curves shown in Figure 3 may be used for the maximum allowable spring stress.

Note : To provide a factor of safety (for both compression and extension springs), the maximum calculated stress should not exceed 85% of the value read off the curve.

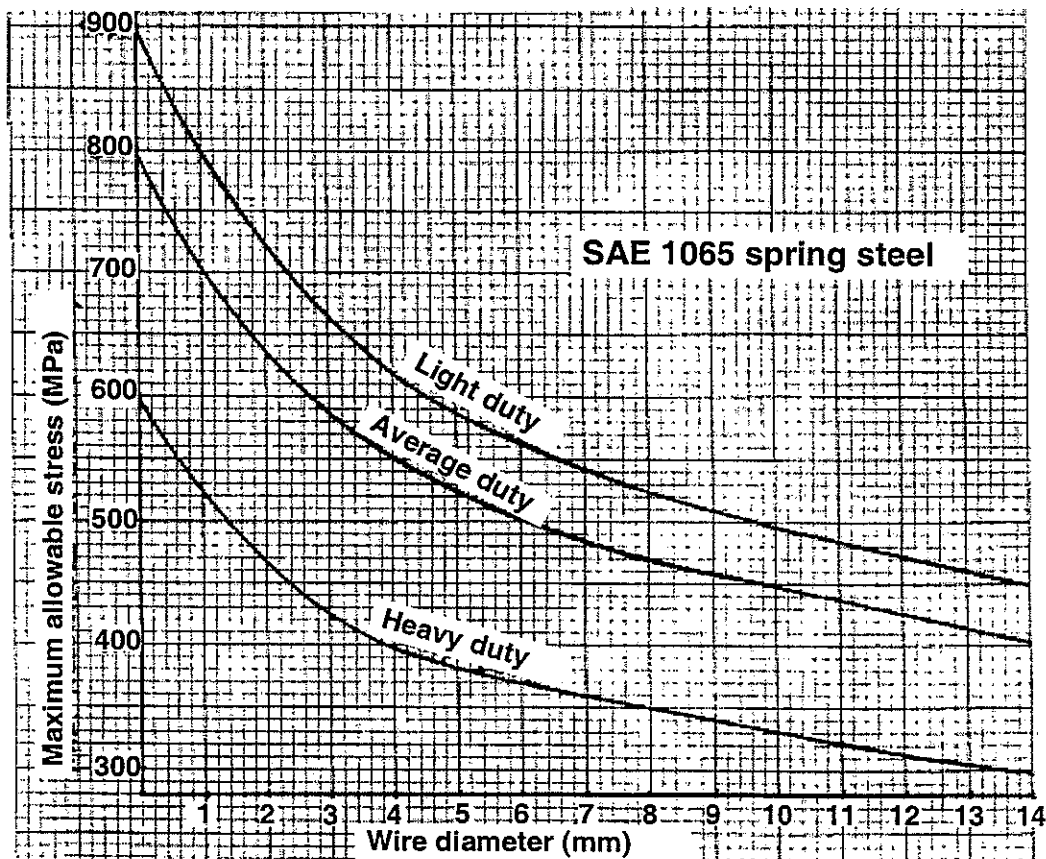


Figure 3 Maximum allowable spring stress

Calculated stress f

Helical springs are stressed in torsional shear + bending.

The torsion shear stress is given by the standard formula:

$$f = \frac{16T}{\pi d^3}$$

Now if force F acts at the centreline of the spring, then the torque T is given by :

$$f = \frac{\bar{F} \cdot D}{2}$$

Substituting in the torsional shear stress formula gives :

$$f = \frac{8 F \cdot D}{\pi d^3}$$

Because there is bending as well as torsion, the combined stress is greater than the torsional shear stress alone. A factor known as the Wahl factor K that has a value greater than 1 is then applied so the final result for the combined stress in the spring is :

$$f = \frac{8 K F D}{\pi d^3} \quad \text{Formula 3}$$

Or in terms of the spring index C :

$$f = \frac{8 K F C}{\pi d^2} \quad \text{Formula 4}$$

Where the Wahl factor K is given by :

$$K = \frac{4 C - 1}{4 C - 4} + \frac{0.615}{C} \quad \text{Formula 5}$$

Notes :

- In these formulas for stress, the stress f is caused by load F (not change in load).
- The spring stress is independent of the number of coils.
- Do not confuse K (Wahl factor) with k (spring constant).

Example 3

A load of 300 N is applied to a compression spring made of spring steel wire of diameter 4 mm. The mean diameter of the spring is 40 mm. Calculate the stress caused by this load.

Solution

$$C = \frac{D}{d} = \frac{40}{4} = 10$$

$$\begin{aligned} K &= \frac{4C - 1}{4C - 4} + \frac{0.615}{C} \\ &= \frac{40 - 1}{40 - 4} + \frac{0.615}{10} \\ &= 1.145 \end{aligned}$$

$$\begin{aligned} f &= \frac{8KC}{\pi d^2} \\ &= \frac{8 \times 1.145 \times 300 \times 10}{\pi \times 4^2} \\ &= 547 \text{ MPa} \end{aligned}$$

Number of coils

It has been seen that the stress in a spring is independent of the number of coils and for the same spring diameter and wire diameter, the stress is the same at the same load regardless as to whether the spring is long or short (large or small number of coils). The number of coils is governed by the deflection required at the given load, the larger the deflection, the greater the number of coils needed. A large deflection also means a small spring constant, so it can also be said that the smaller the spring constant, the more coils required.

The number of coils required can be calculated using the following formula:

$$n = \frac{Gd}{8C^3k} \quad \text{Formula 6}$$

In this formula G = modulus of rigidity of the spring wire. For spring steel wire G is usually taken as 78.6 GPa. Other symbols have meaning as previously defined.

The number of coils n by this formula is the number of *active* coils. For a tension spring, all coils are active and the total number of coils $N = \text{number of active coils}$. Compression springs usually have the two end coils squared and ground or squared (for small diameter wire $< 0.7 \text{ mm}$). In either case, the two end coils sit flat and are inactive (they do not deform under load) so the total number of coils $N = n + 2$.

Note: Whilst it is theoretically possible to have fractional parts of a coil, it is usual to round the number of coils off to the next largest 0.5 of a coil. For example, if the number of coils was calculated to be 5.23, then specify 5.5 coils or if 6.74, specify 7 coils.

Example 4

Calculate the number of coils needed for the spring given in Example 3 if the total deflection caused by the 300 N load is to be 50 mm.

Solution

The spring constant $k = \frac{300}{50} = 6 \text{ N/mm}$

The spring index $C = 10$ and the wire diameter $d = 4 \text{ mm}$

Using $G = 78.6 \text{ GPa}$ and substituting in Formula 6 using mm and MPa units:

$$\begin{aligned} n &= \frac{G d}{8 C^3 k} \\ &= \frac{78.6 \times 10^3 \times 4}{8 \times 10^3 \times 6} \\ &= 6.55 \end{aligned}$$

Hence 7 active coils are needed (6.5 also OK) so the total number of coils = **9** (or 8.5)

Initial length (free length) of a spring L

As shown in the diagram on page 274, extension springs in the no-load position have the coils tightly wound so they are touching one another. The initial length (free length) of the spring between attachment point is then :

$$L = N d + \text{loop lengths} \quad \text{Formula 7}$$

Where the loop lengths depend upon the form of looping provided for attachment of the load or support at each end of the spring.

As shown in the diagram on page 267, compression springs in the no-load position do not have the coils tightly wound so there is a gap between the coils. This space is of course necessary to allow deflection under load.

If a compression spring were compressed solid under load (chock), the length under no-load (free length) would be:

$$L = N d + x_2$$

Where x_2 = total deflection (from zero load position) see Figure 2 on page 255.

However, it is undesirable to have a compression spring chock under load so a clash allowance Ca should be included when determining the free length of the spring.

The clash allowance Ca is the amount by which the design deflection is increased to eliminate the possibility of chocking under load. The clash allowance is usually at least 20% (0.2), however a check of the ASP spring catalogue reveals that their springs usually have a clash allowance of between 30 and 40%.

Using the clash allowance, the free length formula for a compression spring then becomes:

$$L = N d + x_2 (1 + Ca) \quad \text{Formula 8}$$

Example 5

Determine the free length of the compression spring given in Example 4 if a clash allowance of 20% is provided.

Solution

From Example 4: $N = 9$, $d = 4$ mm and max. deflection $x_2 = 50$ mm

$$L = N d + x_2 (1 + C_a)$$

Substituting:

$$\begin{aligned} L &= 9 \times 4 + 50 \times 1.2 \\ &= 36 + 60 \\ &= 96 \text{ mm} \end{aligned}$$

Buckling

As the free length of a compression spring increases in proportion to its diameter, (that is the spring becomes more slender), the spring can buckle under load (in a similar manner to a column).

If $L/d > 10$ the spring will most likely buckle under any load (or deflection).

If $L/d < 10$ the likelihood of buckling depends upon the maximum load (or deflection) and can be determined by reference to the graph shown in Fig. 4 (page 241).

To use this graph, calculate the L/d ratio of the spring and then read off the maximum deflection to free length ratio (x_2/L) for buckling. Then calculate the maximum deflection for buckling x_2 . If the maximum deflection of the spring is less than this, buckling is unlikely. If more than this, buckling is likely and the spring needs to be guided or supported in some way (for example, by fitting the spring over a rod or inside a tube).

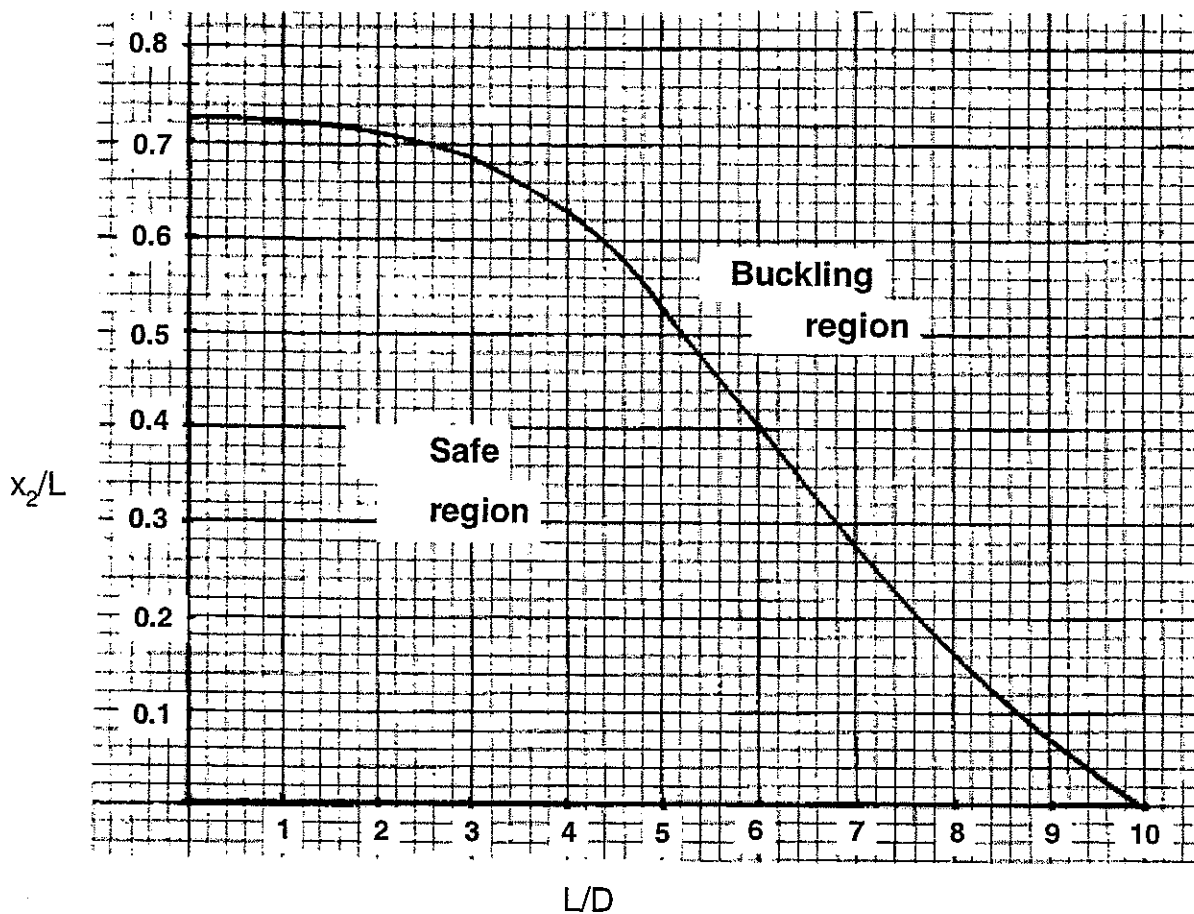


Figure 4 Maximum L/d ratio for buckling

Example 6

Determine if buckling is likely for the spring designed in Example 5 and hence whether or not support is needed.

Solution

$L = 96 \text{ mm}$, $D = 40 \text{ mm}$, $x_2 = 50 \text{ mm}$ (max. defln.)

$\therefore L/d = 2.4$

Using Figure 4, $x_2/L = 0.7$ (max. for buckling).

$\therefore x_2 \text{ max} = 0.7 \times 96 = 67.2 \text{ mm}$

Since x_2 for the spring is 50 mm, buckling is **unlikely** and no support is needed.

Summary - Design procedure for helical springs

The design procedure may vary according to the data given and the constraints imposed but the following summary may be useful:

1. Assume a spring index C using Table 2 page 258 and hence obtain a trial value for the mean diameter D and the wire diameter d . Use the next largest standard wire diameter using Table 1 page 255.
2. Determine the maximum allowable stress (if not given) using Table 3 page 259 to determine the duty and the graph Figure 3 page 259 to determine the maximum allowable stress (assuming SAE 1065 spring steel).
3. Calculate the Wahl factor K using Formula 5 page 260.
4. Calculate the stress in the spring using Formula 3 page 260 or Formula 4 page 260.
5. Compare the calculated stress to the maximum allowable stress. It should be in the range 70-85%. If too high, trial a larger wire diameter and if too small trial a smaller diameter. Repeat steps 1. - 5. until the stress is satisfactory.
6. Determine the spring constant (spring rate) k using Formula 1 page 254 using the maximum force and maximum deflection or the change in force and change in deflection.
7. Determine the number of active coils n using Formula 6 page 261 and hence the total number of coils N .
8. Calculate the free length of the spring L using Formula 7 page 263 (extension spring) or Formula 8 page 263 (compression spring). If the clash allowance is not given assume 20%.
9. If the spring is a compression spring check if buckling is likely by calculating the L/d ratio. If $L/d > 10$ buckling is likely under any load or deflection and if $L/d < 10$ use Fig. 4 page 265. If buckling is likely some guidance or support is needed.
10. Summarise the design preferably with a sketch showing all relevant data.